Infinite hide-and-seek in the space of graphs

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Abstract

For a countable set V, let us view $2^{[V]^2}$ as the space of graphs on the vertex set V. A graph property $\mathcal{S}$ is an isomorphism-invariant subset $\mathcal{S} \subseteq 2^{[V]^2}$. For a fixed vertex set V and a property $\mathcal{S} \subseteq 2^{[V]^2}$, consider the following game. Alternately, Player I (the seeker) plays a pair $\{x, y\} \in [V]^2$, and Player II (the hider) decides whether it is an edge or a non-edge. Player I wins if and only if she can decide whether the graph they are building has property $\mathcal{S}$ *without* playing every pair of V. The graph property $\mathcal{S}$ is called *elusive* if Player I does not have a winning strategy.

For a finite vertex set V, elusive properties were intensively studied in the 70s and 80s. The renowned Aanderaa-Karp-Rosenberg Conjecture, which says that every nontrivial monotone graph property is elusive, has been open since 1973.

In what follows, I am going to talk about elusive properties of *infinite* graphs and share some of our early results. Among others, we proved that connectivity and bipartiteness are elusive but the properties "it contains n independent edges" and "there is no isolated vertex" are non-elusive.

This talk is based on joint work (in progress) with Márton Elekes and Anett Kocsis.